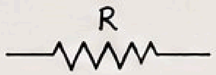
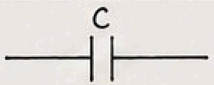
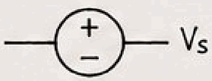
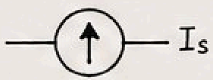
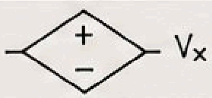
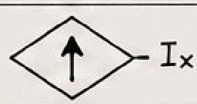
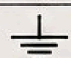
INTRODUCTION TO ELECTRICAL CIRCUIT ANALYSIS

CIRCUIT ELEMENTS

ELEMENT	SYMBOL	VOLTAGE-CURRENT RELATIONSHIP	UNIT	DESCRIPTION
1. Resistor (R)		$v = iR$ (Ohm's Law)	Ohm (Ω)	Opposes the flow of current Dissipates energy
2. Inductor (L)		$v = L \frac{di}{dt}$	Henry (H)	Stores energy in magnetic field
3. Capacitor (C)		$i = C \frac{dv}{dt}$	Farad (F)	Stores energy in electric field
4. Independent Voltage Source		$v = V_s$ (given) (Independent of i)	Volt (V)	Maintains a specific voltage
5. Independent Current Source		$i = I_s$ (given) (Independent of v)	Ampere (A)	Maintains a specific current
6. Dependent Voltage Source		$v = \mu V_x$ (μ: Voltage gain)	Volt (V)	Voltage controlled voltage source (VCVS)
7. Dependent Current Source		$i = \beta I_x$ (β: Current gain)	Ampere (A)	Current controlled current source (CCCS)
8. Ground		$v = 0$	–	Reference node (0 V)
9. Switch (Open)		$i = 0$	–	No conduction path
10. Switch (Closed)		$v = 0$	–	Complete conduction path

PASSIVE ELEMENTS

1. Resistor (R)
2. Inductor (L)
3. Capacitor (C)

} Do not supply energy. They absorb or store energy.

ACTIVE ELEMENTS

1. Independent Sources
2. Dependent Sources

} Can supply energy to the circuit.

Note:

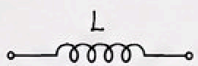
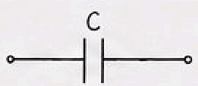
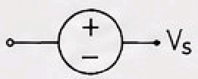
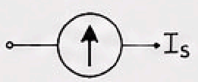
* Current enters the positive terminal of an element.
* All elements are ideal unless specified.

LINEAR AND NON-LINEAR DEVICES

Electrical devices are basic building blocks of circuits. They can be classified as Linear or Non-Linear depending on the relationship between voltage and current.

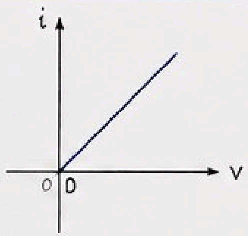
1. LINEAR DEVICES

A device is linear if the voltage-current relationship is linear and it obeys superposition principle.

Device	Symbol	V-I Relationship	Notes
1. Resistor (R)		$V = IR$ (Ohm's Law)	$V \propto I$ Straight line through origin
2. Inductor (L)		$v = L \frac{di}{dt}$	Linear element (energy stored in magnetic field)
3. Capacitor (C)		$i = C \frac{dv}{dt}$	Linear element (energy stored in electric field)
4. Independent Voltage Source (V_s)		$V = V_s$ (constant)	Ideal source, value is constant
5. Independent Current Source (I_s)		$i = I_s$ (constant)	Ideal source, value is constant

Properties of Linear Devices

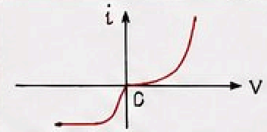
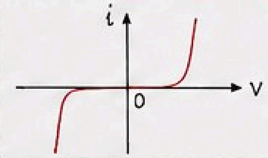
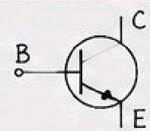
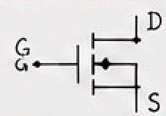
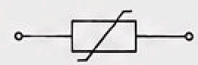
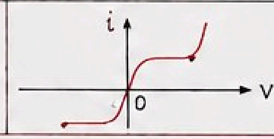
- Obey superposition principle.
- V-I relationship is linear.
- Parameters (R, L, C) are constant.
- Examples: R, L, C, independent sources.



Linear V-I characteristic

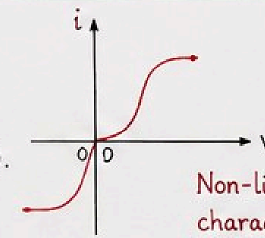
2. NON-LINEAR DEVICES

A device is non-linear if the voltage-current relationship is not linear and it does not obey superposition principle.

Device	Symbol	V-I Relationship	Notes
1. Diode			Allows current in one direction only. Non-linear V-I characteristic.
2. Zener Diode			Operates in forward bias and reverse breakdown region. Non-linear.
3. Transistor (BJT)		$i_C = f(i_B, V_{CE})$	Relationship between currents and voltages is non-linear.
4. MOSFET		$i_D = f(V_{GS}, V_{DS})$	Non-linear device used for switching and amplification.
5. Varistor			Voltage dependent resistor. Non-linear.

Properties of Non-Linear Devices

- Do not obey superposition principle.
- V-I relationship is non-linear.
- Parameters are not constant (vary with voltage/current).
- Examples: Diode, Transistor, MOSFET, Varistor, etc.



Non-linear V-I characteristic

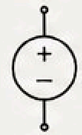
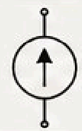
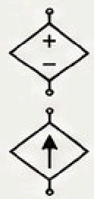
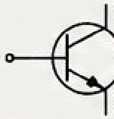
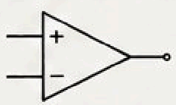
Note: Most practical circuits use a combination of linear and non-linear devices.

ACTIVE AND PASSIVE DEVICES

Circuit elements are classified as Active or Passive depending on their ability to supply energy to a circuit.

1. ACTIVE DEVICES

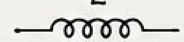
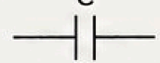
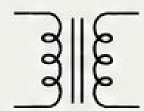
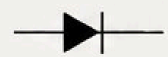
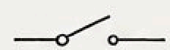
Active devices can supply energy or control the flow of energy in a circuit.

Device	Symbol	Description / Function	Example
1. Independent Voltage Source	 V_s	Provides a specified voltage regardless of the current.	Battery, DC source
2. Independent Current Source	 I_s	Provides a specified current regardless of the voltage.	Current Generator
3. Dependent (Controlled) Source	 V_x, I_x	Source whose value depends on some other voltage or current in the circuit.	VCVS, VCCS, CCVS, CCCS
4. Transistor (BJT)	 C, E	Used for amplification and switching. Controls large current with small input.	2N3904, BC547
5. Operational Amplifier (Op-Amp)		Provides high gain; used in amplifying, filtering, comparing, etc.	741, LM358

Key Point: Active devices can deliver power to a circuit.

2. PASSIVE DEVICES

Passive devices cannot supply energy; they only store or dissipate energy.

Device	Symbol	Description / Function	Example
1. Resistor (R)	R 	Limits current and dissipates energy in the form of heat.	Carbon resistor, Metal film resistor
2. Inductor (L)	L 	Stores energy in magnetic field.	Choke, Coil, Inductor
3. Capacitor (C)	C 	Stores energy in electric field.	Ceramic, Electrolytic, Film capacitor
4. Transformer		Transfers AC energy between circuits through mutual induction.	Step-up, Step-down Transformer
5. Diode		Allows current to flow in one direction only.	1N4148, 1N4007
6. Switch		Connects or disconnects parts of a circuit.	Toggle switch, Push button switch

Key Point: Passive devices cannot deliver power; they only store or dissipate energy.

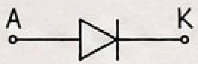
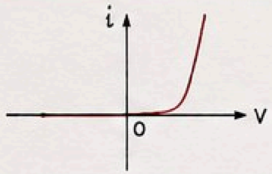
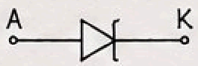
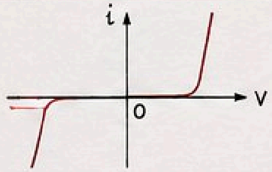
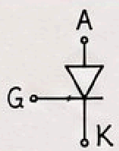
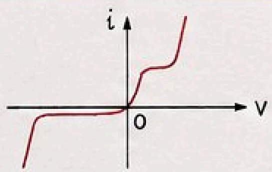
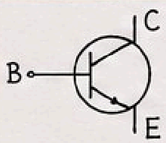
Summary: Active devices → Can supply energy or control energy flow. (Sources, Transistors, Op-Amps, etc.)
Passive devices → Cannot supply energy; they store or dissipate energy. (R, L, C, Transformer, Diode, Switch, etc.)

Unilateral and Bilateral Devices

Electrical devices are classified as Unilateral or Bilateral depending on the symmetry of their V-I characteristics and the ability to conduct current in one or both directions.

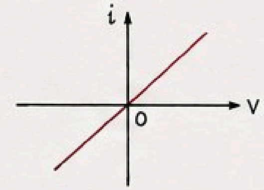
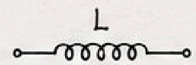
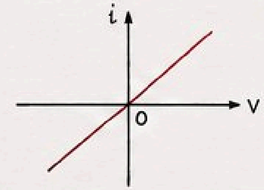
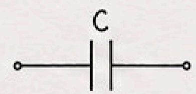
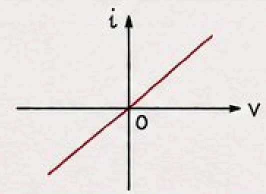
1. Unilateral Devices

Devices which allow current to flow primarily in one direction are called unilateral devices.

Device	Symbol	V-I Characteristic	Description / Example
1. Diode			Allows current in forward direction only. Ex: PN junction diode
2. Zener Diode			Allows forward current and reverse breakdown beyond certain voltage. Ex: Zener diode
3. SCR (Silicon Controlled Rectifier)			Conducts in forward direction when gate is triggered. Ex: SCR
4. Transistor (BJT)		Non-linear and unilateral.	Current flow controlled in one direction between collector and emitter. Ex: BJT

2. Bilateral Devices

Devices which allow current to flow in both directions are called bilateral devices.

Device	Symbol	V-I Characteristic	Description / Example
1. Resistor (R)			Allows current in both directions. V-I characteristic is linear. Ex: Carbon resistor
2. Inductor (L)			Current can flow in either direction. Stores energy in magnetic field. Ex: Choke, Coil
3. Capacitor (C)			Current can flow in either direction. Stores energy in electric field. Ex: Ceramic capacitor
4. Transmission Line		Linear and bilateral.	Allows voltage and current in both directions. Ex: Two-wire line

Key Point (Unilateral): Unilateral devices conduct current mainly in one direction and block it in the opposite direction.

Key Point (Bilateral): Bilateral devices conduct current in both directions and their V-I characteristics are symmetrical.

* Note: Unilateral devices are used in rectification, switching, and control applications.
Bilateral devices are used in signal processing, energy storage, and passive networks.

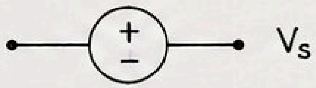
ENERGY SOURCES

Energy sources are the circuit elements that supply energy to the circuit.
There are two types of ideal energy sources.

1. VOLTAGE SOURCE

* A voltage source maintains a constant voltage across its terminals irrespective of the current flowing through it.

(a) Symbol



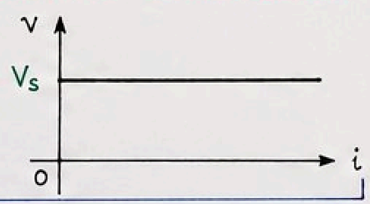
(b) V-I Relationship

Voltage is constant.

$v = V_s$ (constant)

Current i can be any value depending on the external circuit.

(c) Terminal Characteristic



The terminal voltage remains constant for any value of current.

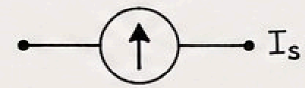
(d) Examples

- Battery
- DC Power Supply
- Generator (constant voltage)

2. CURRENT SOURCE

* A current source maintains a constant current through its terminals irrespective of the voltage across it.

(a) Symbol



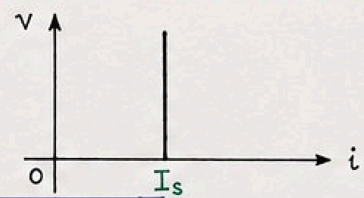
(b) V-I Relationship

Current is constant.

$i = I_s$ (constant)

Voltage v can be any value depending on the external circuit.

(c) Terminal Characteristic



The terminal current remains constant for any value of voltage.

(d) Examples

- Current Generator
- Photocell (approx.)
- Transistor current source
- Solar Cell (approx.)

Key Point :

* Voltage Source → Specifies voltage (V) and current (I) is the result.
* Current Source → Specifies current (I) and voltage (V) is the result.

Note :

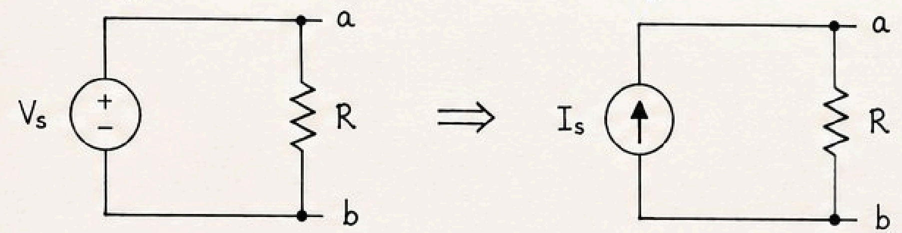
In practical sources, internal resistances exist but in circuit analysis we use ideal sources.

Source Transformation

Source transformation is the process of converting a voltage source into an equivalent current source or a current source into an equivalent voltage source without changing the external behavior (V-I relationship) of the circuit.

1. Voltage Source to Current Source

Original Circuit — Equivalent Circuit



Relationship between sources

$$I_s = \frac{V_s}{R} \quad \text{[Current in amperes]}$$

R remains the same.

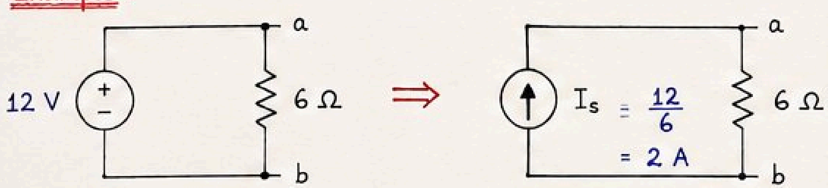
Derivation

In the original circuit,

$$I = \frac{V_s}{R}$$

⇒ This current I is provided by the current source I_s.

Example :



Verification :

* Open circuit (a-b open):

$$V_{ab} = V_s = I_s R = 12 \text{ V}$$

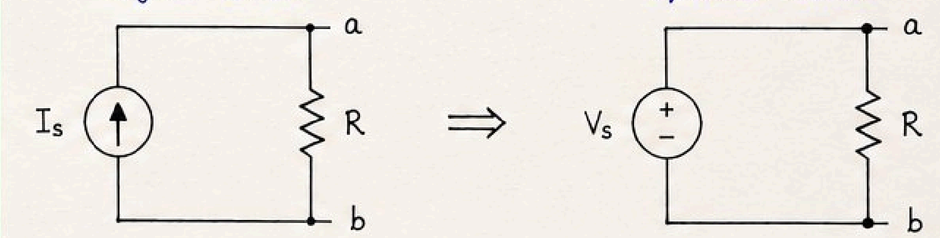
* Short circuit (a-b shorted):

$$I_{sc} = \frac{V_s}{R} = I_s = 2 \text{ A}$$

Since both circuits give same V_{ab} and I_{sc}, they are equivalent.

2. Current Source to Voltage Source

Original Circuit — Equivalent Circuit



Relationship between sources

$$V_s = I_s R \quad \text{[Voltage in volts]}$$

R remains the same.

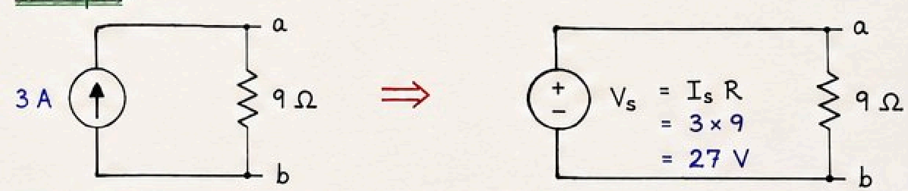
Derivation

In the original circuit,

$$V = I R = I_s R$$

⇒ This voltage is provided by the voltage source V_s.

Example :



Verification :

* Open circuit (a-b open):

$$V_{ab} = V_s = I_s R = 27 \text{ V}$$

* Short circuit (a-b shorted):

$$I_{sc} = \frac{V_s}{R} = I_s = 3 \text{ A}$$

Since both circuits give same V_{ab} and I_{sc}, they are equivalent.

Key Point : A practical circuit can be simplified by transforming sources to make analysis easier.

[Source transformation changes the source type but not the external V-I behavior of the circuit.]

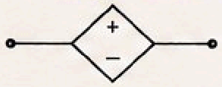
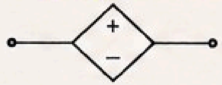
Analysis with Dependent Current and Voltage Sources

Dependent (controlled) sources are those whose value depends on some other voltage or current elsewhere in the circuit.

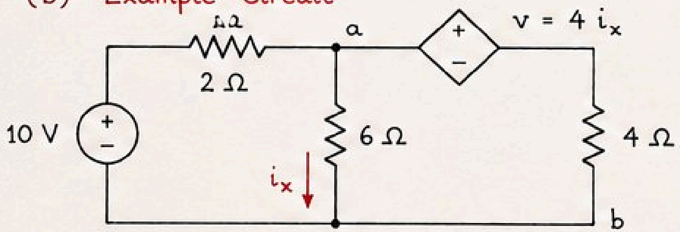
1. Analysis with Dependent Voltage Sources (DVS)

A dependent voltage source is a voltage source whose voltage is proportional to some voltage or current in the circuit.

(a) Types and Symbols

Voltage Controlled Voltage Source (VCVS)	Current Controlled Voltage Source (CCVS)
 $v = \mu v_x$	 $v = r_m i_x$

(b) Example Circuit



Controlling variable: i_x = current through 6 Ω resistor

(c) Analysis (Nodal Method)

Let node b be reference (0 V) and node a has voltage V_a.

Current $i_x = \frac{V_a}{6}$

Voltage across dependent source,

$v = 4 i_x = 4\left(\frac{V_a}{6}\right) = \frac{2}{3} V_a$ [+ at node a side]

Apply KCL at node a:

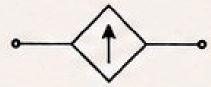
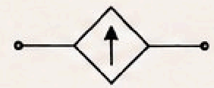
$\frac{V_a - 10}{2} + \frac{V_a}{6} + \frac{V_a - v}{4} = 0$ [(Node a is connected to three branches)

Substitute $v = \frac{2}{3} V_a$ and solve for V_a.

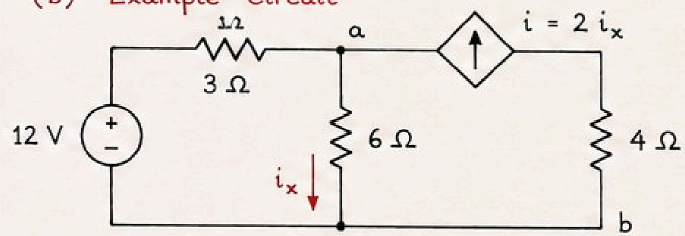
2. Analysis with Dependent Current Sources (DCS)

A dependent current source is a current source whose current is proportional to some voltage or current in the circuit.

(a) Types and Symbols

Voltage Controlled Current Source (VCCS)	Current Controlled Current Source (CCCS)
 $i = g v_x$	 $i = \beta i_x$

(b) Example Circuit



Controlling variable: i_x = current through 6 Ω resistor

(c) Analysis (Nodal Method)

Let node b be reference (0 V) and node a has voltage V_a.

$i = V_x)/6$

Current of dependent source, $i = 2 i_x = 2\left(\frac{V_a}{6}\right) = \frac{V_a}{3}$ [from node b to a)

Apply KCL at node a:

$\frac{V_a - 12}{3} + \frac{V_a}{6} - i = 0$ [−i because current source enters node a)

Substitute $i = \frac{V_a}{3}$ and solve for V_a.

Key Points

- ✱ Dependent sources make the circuit equations coupled.
- ✱ Use KCL, KVL, nodal or mesh methods including the dependent source relationships.
- ✱ Always define the controlling variable and its direction.

✱ VCVS: $v = \mu v_x$, CCVS: $v = r_m i_x$

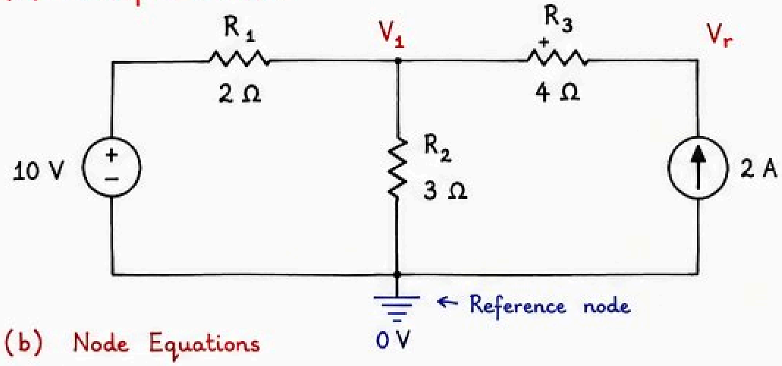
✱ VCCS: $i = g v_x$, CCCS: $i = \beta i_x$

(where μ, r_m, g and β are constants)

1. NODE ANALYSIS (KCL Method)

It is a method of circuit analysis in which node voltages (with respect to a reference node) are the unknowns.

(a) Example Circuit



Data :

$R_1 = 2\ \Omega$

$R_2 = 3\ \Omega$

$R_3 = 4\ \Omega$

$V_s = 10\ V$ (left)

$I_s = 2\ A$ (upward)

(b) Node Equations

At right node V_r : The current source injects 2 A into the node V_r.

Hence, current through $R_3 = 2\ A \Rightarrow \dfrac{V_r - V_1}{4} = 2$

$$\boxed{V_r = V_1 + 8}$$

At node V_1 : Apply KCL (currents leaving the node)

$$\frac{V_1 - 10}{2} + \frac{V_1}{3} + \frac{V_1 - V_r}{4} = 0 \qquad [V_r = V_1 + 8]$$

Substitute V_r : $\dfrac{V_1 - 10}{2} + \dfrac{V_1}{3} + \dfrac{(V_1 - V_1)}{4} = 0$

$$\frac{V_1 - 10}{2} + \frac{V_1}{3} - 2 = 0$$

Multiply by 6 $\Rightarrow 3(V_1 - 10) + 2V_1 - 12 = 0 \Rightarrow 5V_1 = 42$

$$\boxed{V_1 = \frac{42}{5} = 8.4\ V}$$

Key Point :

Do not write $\dfrac{V_1}{4} - 2$. It is incorrect because the current in R_3 depends on $(V_r - V_1)$, not V_1 alone.

(c) Other Node Voltages / Branch Currents

$$\boxed{V_r = V_1 + 8 = 8.4 + 8 = 16.4\ V}$$

- Current in R_2 (downward) : $I_{R2} = \dfrac{V_1}{R_2} = \dfrac{8.4}{3} = 2.8\ A$
- Current in R_1 (from 10 V source to node) : $I_{R1} = \dfrac{10 - V_1}{2} = \dfrac{10 - 8.4}{2} = 0.8\ A$
- Current in R_3 (from node to current source) : $I_{R3} = 2\ A$ (to the right)

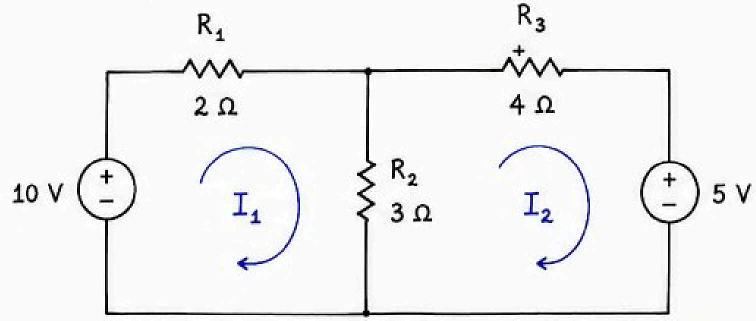
Key Points (Node Analysis)

* Choose a reference (ground) node.
* Assign node voltages to the remaining nodes.
* Apply KCL at each non-reference node.
* Solve simultaneous equations to get node voltages.
* Branch currents can be found using Ohm's law.

2. MESH ANALYSIS (KVL Method)

It is a method of circuit analysis in which mesh currents are the unknowns.

(a) Example Circuit



Data :

$R_1 = 2\ \Omega$

$R_2 = 3\ \Omega$

$R_3 = 4\ \Omega$

$V_1 = 10\ V$ (left)

$V_2 = 5\ V$ (right)

(b) Mesh Equations (Apply KVL)

Mesh 1 (clockwise)

$$-10 + 2I_1 + 3(I_1 - I_2) = 0$$

$$\Rightarrow \boxed{5I_1 - 3I_2 = 10 \quad \text{— (1)}}$$

Mesh 2 (clockwise)

$$-5 + 4I_2 + 3(I_2 - I_1) = 0$$

$$\Rightarrow \boxed{-3I_1 + 7I_2 = 5 \quad \text{— (2)}}$$

(c) Solving Equations (1) and (2)

From (1) : $5I_1 - 3I_2 = 10 \Rightarrow I_1 = \dfrac{10 + 3I_2}{5}$

Substitute in (2) : $-3\left(\dfrac{10 + 3I_2}{5}\right) + 7I_2 = 5$

$$\Rightarrow \frac{-30 - 9I_2 + 35I_2}{5} = 5 \Rightarrow \frac{26I_2}{5} = 5 \Rightarrow I_2 = 2.5\ A$$

Then, $I_1 = \dfrac{10 + 3 \times 2.5}{5} = \dfrac{17.5}{5} = 3.5\ A$

$$\boxed{\begin{array}{l} I_1 = 3.5\ A \\ I_2 = 2.5\ A \end{array}}$$

(d) Branch Currents

Current in $R_1 = I_1 = 3.5\ A$ (left to right)

Current in $R_3 = I_2 = 2.5\ A$ (left to right)

Current in $R_2 = I_1 - I_2 = 3.5 - 2.5 = 1.0\ A$ (downward)

Key Points (Mesh Analysis)

* Identify meshes (no current sources between two elements; if present, use supermesh).
* Assign mesh currents (usually clockwise).
* Apply KVL around each mesh.
* Solve simultaneous equations to find mesh currents.
* Find branch currents using mesh currents.

Concept of Duality and Dual Networks

1. Concept of Duality

* In electrical network theory, every circuit has a dual circuit obtained by interchanging certain circuit quantities and elements.
* If the equations of a network remain valid for the interchanged (dual) quantities, the two networks are called duals of each other.
* Duality helps in simplifying the analysis and in deriving new circuits from known circuits.

Principle of Duality

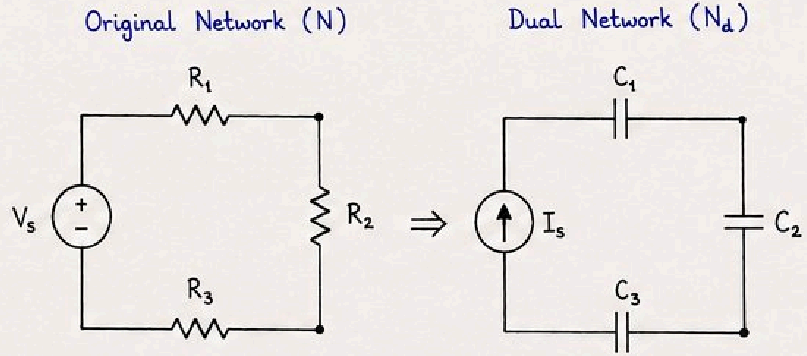
If a statement is true for a network, the dual of that statement is also true for the dual network obtained by interchanging the dual pairs.

2. Dual Pair of Quantities and Elements

Original Quantity / Element		Dual Quantity / Element
Voltage (V)	⟷	Current (I)
Current (I)	⟷	Voltage (V)
Resistance (R)	⟷	Conductance ($G = 1/R$)
Inductance (L)	⟷	Capacitance (C)
Series Connection	⟷	Parallel Connection
Parallel Connection	⟷	Series Connection
KVL (loop law)	⟷	KCL (node law)
Mesh	⟷	Node
Loop	⟷	Cut-set
Voltage Source	⟷	Current Source
Open Circuit	⟷	Short Circuit
Short Circuit	⟷	Open Circuit

3. Dual Networks

- Two networks are duals if one can be obtained from the other by interchanging all the dual pairs.

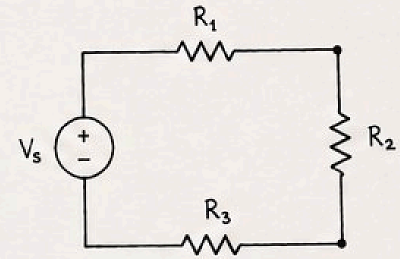


Observations:

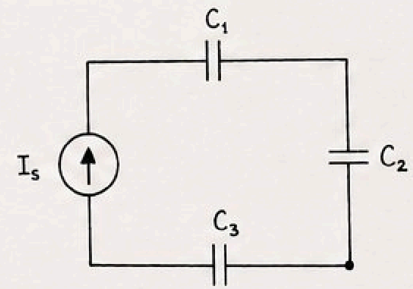
- Voltage source V_s in N ⟷ Current source I_s in N_d
- Resistors R_1, R_2, R_3 in N ⟷ Capacitors C_1, C_2, C_3 in N_d
- Series in N ⟷ Parallel in N_d
- KVL in N ⟷ KCL in N_d
- Mesh in N ⟷ Node in N_d

4. Example

- Consider the network N shown below.



- The dual network N_d is obtained as:



Where,
$C_1 \leftrightarrow R_1$, $C_2 \leftrightarrow R_2$, $C_3 \leftrightarrow R_3$
$I_s \leftrightarrow V_s$

5. Importance / Uses

* Duality provides a powerful tool to obtain a new network from a given network.
* It helps in checking the correctness of network equations.
* It simplifies the analysis of complex circuits.
* Many theorems have their duals (e.g., Thevenin ⟷ Norton).

6. Important Points

* Dual of dual is the original network.
* Not every practical network has a physically realizable dual (due to practical constraints).
* Duality holds for linear, bilateral networks.

Key Takeaway: Duality establishes a one-to-one correspondence between two networks by interchanging voltage ⟷ current, series ⟷ parallel, KVL ⟷ KCL and the associated elements.