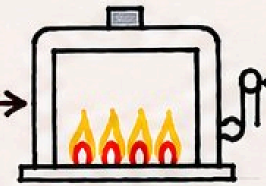
Industrial Control - Examples

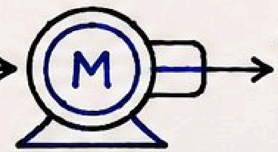
Industrial control systems are used to regulate and maintain the desired performance of machines, processes and plants.

1. **Temperature Control**
   Used in furnaces, ovens, boilers, chemical reactors. →



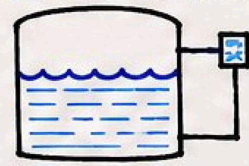
→ **Variable controlled:** Temperature (°C)

2. **Speed Control of DC Motor**
   Used in conveyor belts, robotics, machine tools, fans, pumps. →



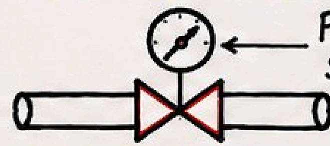
→ Speed (RPM)

3. **Level Control in Storage Tank**
   Used in water tanks, oil tanks, chemical storage tanks. →



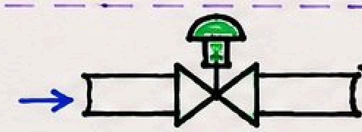
→ Liquid Level

4. **Pressure Control in Pipes**
   Used in gas pipelines, steam systems, hydraulic systems. →



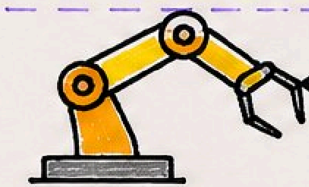
→ Pressure (bar)

5. **Flow Control**
   Used in water supply, chemical dosing, cooling systems. →



→ Flow Rate (L/min)

6. **Position Control in Robotics**
   Used in robotic arms, CNC machines, camera positioning systems. →

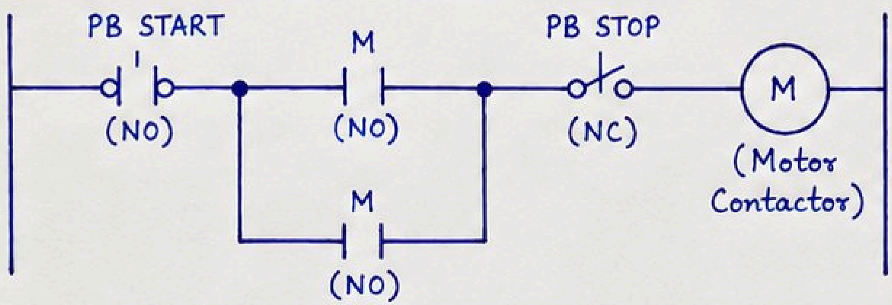


→ Position / Angle (degrees)

* **Common components:** Sensors → Controller → Actuator → Process (Plant) → Output

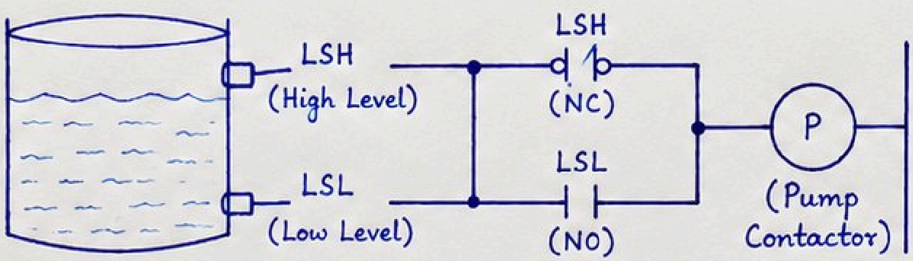
Industrial Control Examples

1. Motor Start / Stop Control



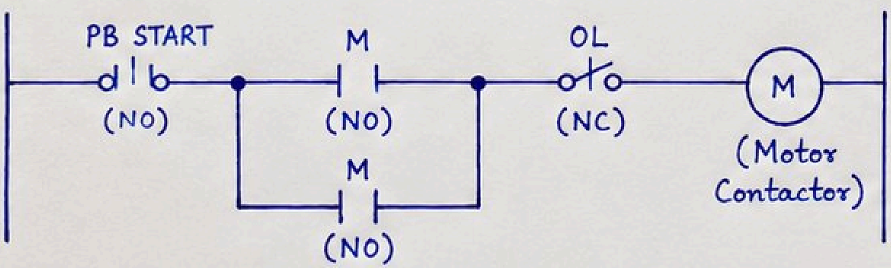
- Press START (NO) → M coil energizes.
- M (NO) contact closes and seals in (holds) the circuit.
- Press STOP (NC) → M coil de-energizes and motor stops.
- Used to start and stop a motor from pushbuttons.

2. Tank Level Control (Automatic Pump Control)



- When level falls to low level, LSL (NO) closes.
- When level rises to high level, LSH (NC) opens.
- Pump runs when level is low and stops when level is high.
- Maintains liquid level between low and high limits.

3. Conveyor Motor with Overload Protection

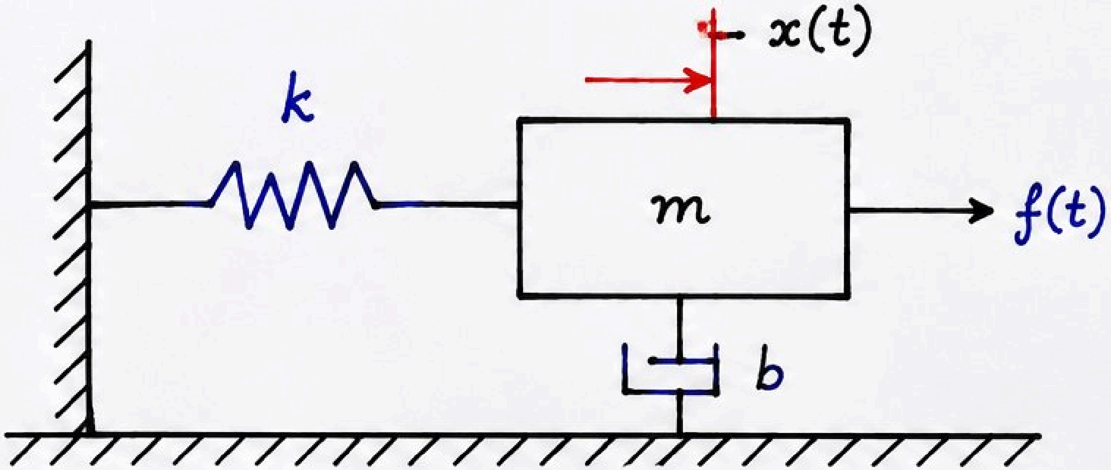


- Press START → M coil energizes and conveyor motor starts.
- OL (Overload Relay) NC contact opens on overload → M coil de-energizes.
- Protects motor from damage due to overload.

Note: NO = Normally Open Contact	M = Motor Contactor Coil	LSH = Level Switch High
NC = Normally Closed Contact	P = Pump Contactor Coil	LSL = Level Switch Low
		OL = Overload Relay

Mathematical Modeling of Physical Systems

1. Mass – Spring – Damper System



m : mass (kg)

b : damping coefficient (N·s/m)

k : spring constant (N/m)

$f(t)$: external force (N)

$x(t)$: displacement (m)

Applying Newton's Second Law,

Sum of forces = mass × acceleration

$$m\,\ddot{x}(t) = -b\,\dot{x}(t) - k\,x(t) + f(t)$$

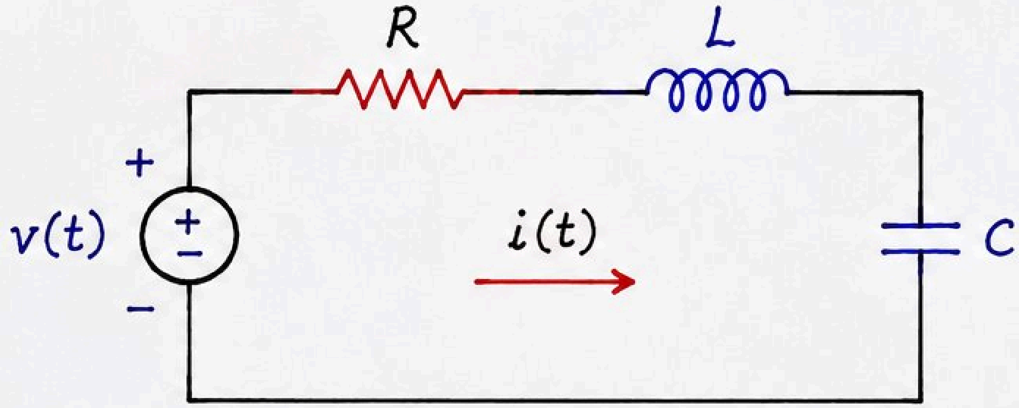
$$\boxed{m\,\ddot{x}(t) + b\,\dot{x}(t) + k\,x(t) = f(t)}$$ → Differential Equation

Transfer Function :

$$\boxed{G(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k}}$$

Mathematical Modeling of Physical Systems

2. Electrical RLC Circuit



$v(t)$: input voltage (V)

$i(t)$: current (A)

By Kirchhoff's Voltage Law (KVL),

$$v(t) = v_R(t) + v_L(t) + v_C(t)$$

$$v(t) = R\,i(t) + L\,\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt$$

Differentiating w.r.t. time,

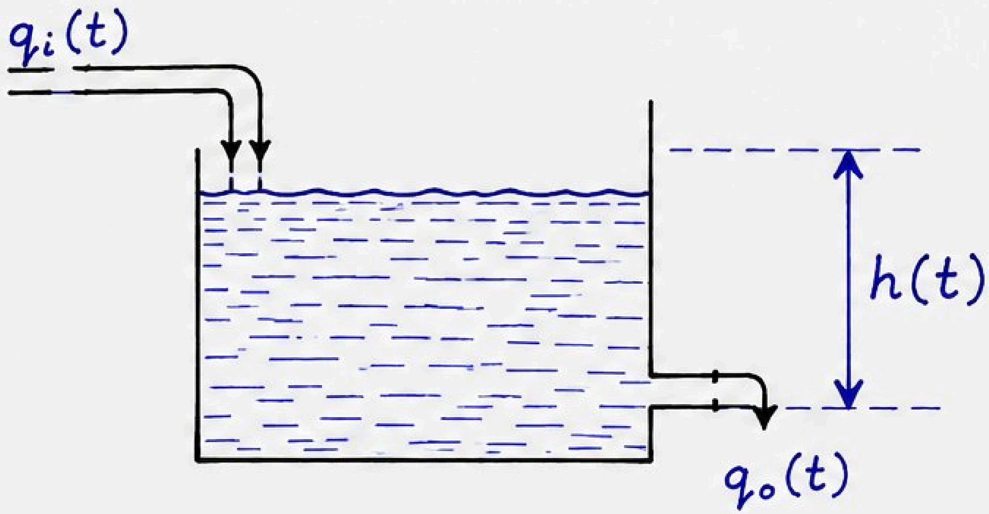
$$\boxed{L\,\frac{d^2 i(t)}{dt^2} + R\,\frac{di(t)}{dt} + \frac{1}{C}\,i(t) = \frac{dv(t)}{dt}}$$ → Differential Equation

Transfer Function :

$$\boxed{G(s) = \frac{I(s)}{V(s)} = \frac{1}{Ls^2 + Rs + \frac{1}{C}}}$$

Mathematical Modeling of Physical Systems

3. Tank Level System



A : cross-sectional area (m^2)

$h(t)$: liquid level (m)

$q_i(t)$: inlet flow rate (m^3/s)

$q_o(t)$: outlet flow rate (m^3/s)

Assuming outlet through an orifice,

$$q_o(t) = \frac{1}{R} h(t) \qquad (R : \text{flow resistance})$$

Mass balance :

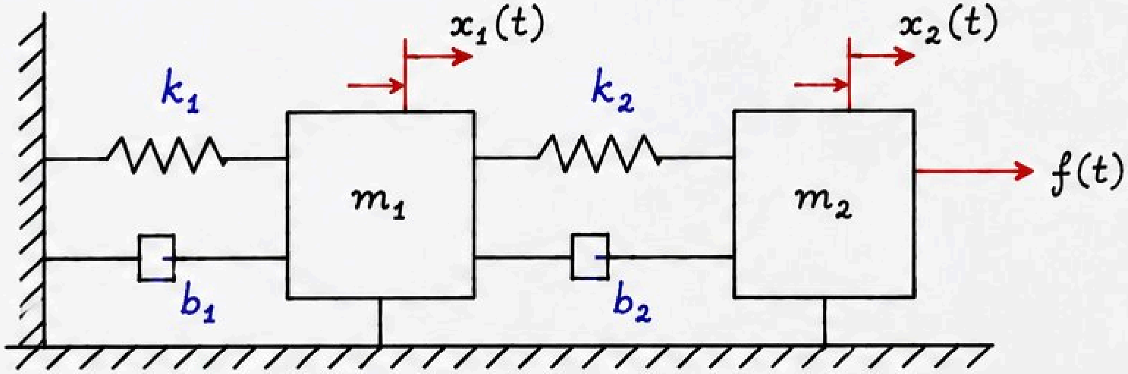
$$\boxed{A \frac{dh(t)}{dt} = q_i(t) - q_o(t) = q_i(t) - \frac{1}{R} h(t)} \rightarrow \text{Differential Equation}$$

Transfer Function :

$$\boxed{G(s) = \frac{H(s)}{Q_i(s)} = \frac{R}{ARs + 1}}$$

Mathematical Modeling of Physical Systems

4. Two-Mass – Spring – Damper System



m_1, m_2 : mases (kg)

k_1, k_2 : spring constants (N/m)

b_1, b_2 : damping coefficients (N·s/m)

$x_1(t), x_2(t)$: displacements (m)

$f(t)$: external force on m_2 (N)

Applying Newton's Second Law,

$$m_1 \ddot{x}_1(t) = -b_1 \dot{x}_1(t) - k_1 x_1(t) - b_2[\dot{x}_1(t) - \dot{x}_2(t)] - k_2[x_1(t) - x_2(t)] \quad -(1)$$

$$m_2 \ddot{x}_2(t) = -b_2[\dot{x}_2(t) - \dot{x}_1(t)] - k_2[x_2(t) - x_1(t)] + f(t) \quad -(2)$$

In Matrix Form,

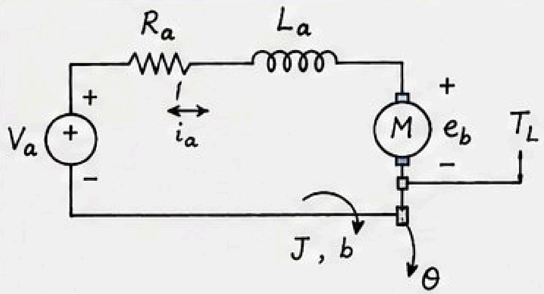
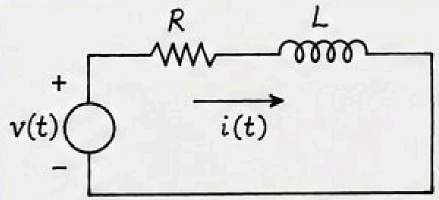
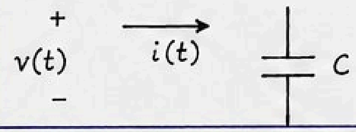
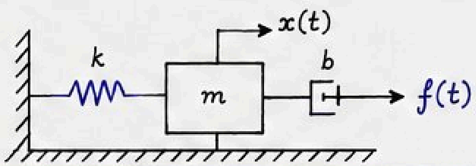
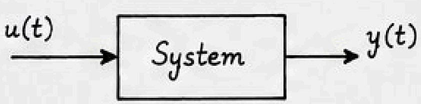
$$\boxed{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{Bmatrix} + \begin{bmatrix} b_1 + b_2 & -b_2 \\ -b_2 & b_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} = \begin{Bmatrix} 0 \\ f(t) \end{Bmatrix}} \rightarrow \text{Differential Equation}$$

Transfer Function : (Force $f(t)$ acting on m_2, output $x_2(t)$)

$$\boxed{G(s) = \frac{X_2(s)}{F(s)} = \frac{b_2 s + k_2}{[m_1 s^2 + (b_1 + b_2)s + (k_1 + k_2)](m_2 s^2 + b_2 s + k_2) - (b_2 s + k_2)^2}}$$

CONTROL HARDWARES AND THEIR MODELS

In control systems, various physical hardware components are represented by mathematical models to analyze and design the controller.

S. No.	Hardware	Physical Representation	Mathematical Model (Differential Equation)	Transfer Function
1.	DC Motor (Armature Controlled)		Electrical : $L_a \frac{di_a}{dt} + R_a i_a + e_b = v_a$ $e_b = K_b \frac{d\theta}{dt}$ Mechanical : $J \frac{d^2\theta}{dt^2} + b \frac{d\theta}{dt} = K_t i_a - T_L$	$\frac{\Theta(s)}{V_a(s)} = \frac{K_t}{(L_a s + R_a)(Js^2 + bs) + K_t K_b}$
2.	R − L Circuit		By KVL, $v(t) = R\,i(t) + L \frac{di(t)}{dt}$ or, $L \frac{di(t)}{dt} + R\,i(t) = v(t)$	$\frac{I(s)}{V(s)} = \frac{1}{Ls + R}$
3.	Capacitor		$i(t) = C \frac{dv(t)}{dt}$	$\frac{I(s)}{V(s)} = Cs$
4.	Mass − Spring − Damper System		By Newton's Second Law, $m \frac{d^2x(t)}{dt^2} + b \frac{dx(t)}{dt} + kx(t) = f(t)$	$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k}$
5.	First Order System (Generic)		$\tau \frac{dy(t)}{dt} + y(t) = K u(t)$ where, τ = time constant , K = gain	$\frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1}$

Note : These models are linear time-invariant (LTI) models commonly used in control system analysis and controller design.

Transfer Function of LTI Systems

The transfer function of a Linear Time-Invariant (LTI) system is defined as the ratio of the Laplace transform of the output to the Laplace transform of the input, with all initial conditions assumed to be zero.

$u(t)$ (Input) $\longrightarrow$ LTI System $\longrightarrow$ $y(t)$ (Output)

Transfer Function

$$G(s) = \frac{Y(s)}{U(s)}$$

where,

$U(s)$ = Laplace transform of $u(t)$

$Y(s)$ = Laplace transform of $y(t)$

From Differential Equation:

Consider the differential equation of an LTI system

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_1 \frac{dy(t)}{dt} + a_0 y(t)$$

$$= b_m \frac{d^m u(t)}{dt^m} + b_{m-1} \frac{d^{m-1} u(t)}{dt^{m-1}} + \cdots + b_1 \frac{du(t)}{dt} + b_0 u(t)$$

Taking Laplace transform on both sides with zero initial conditions,

$$(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0) Y(s)$$

$$= (b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0) U(s)$$

Therefore,

$$\boxed{G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}}$$

Examples:

1. First Order System

$$\tau \frac{dy(t)}{dt} + y(t) = K u(t)$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1}$$

2. Second Order System

$$\frac{d^2 y(t)}{dt^2} + 2\zeta\omega_n \frac{dy(t)}{dt} + \omega_n^2 y(t) = K u(t)$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Properties:

1. $G(s)$ is independent of initial conditions.
2. $G(s)$ completely characterizes the output response for any input.
3. Poles = roots of denominator ; Zeros = roots of numerator.

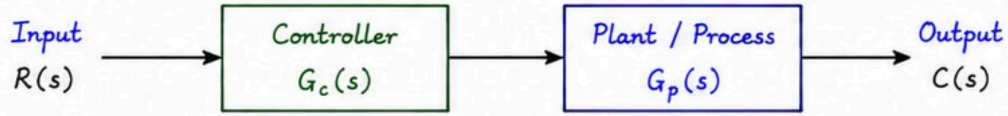
Note: Transfer function is applicable only for linear time-invariant systems.

Feedback Control : Open Loop and Closed Loop Systems

In control systems, the relationship between input and output can be established in two ways :

(i) Open loop control system (ii) Closed loop control system (Feedback control system)

1. Open Loop Control System



Overall Transfer Function :

$$\boxed{\frac{C(s)}{R(s)} = G_{OL}(s) = G_c(s)\,G_p(s)}$$

where, $G_c(s)$ = Controller transfer function

$G_p(s)$ = Plant transfer function

$G_{OL}(s)$ = Open loop transfer function

Characteristics :

- The output does not have any effect on the input.
- No feedback path is present.

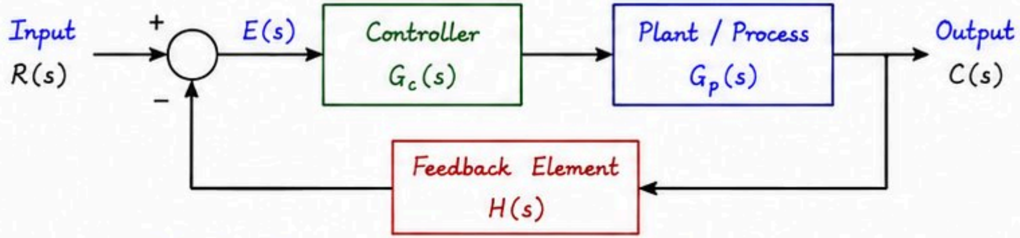
Advantages :

- Simple and inexpensive
- Easy to design
- Generally stable

Disadvantages :

- Inaccurate
- Does not correct errors
- Sensitive to disturbances
- Calibration required

2. Closed Loop (Feedback) Control System



Overall Transfer Function (with negative feedback) :

$$\boxed{\frac{C(s)}{R(s)} = G_{CL}(s) = \frac{G_c(s)\,G_p(s)}{1 + G_c(s)\,G_p(s)\,H(s)}}$$

where, $H(s)$ = Feedback element transfer function

$G_{CL}(s)$ = Closed loop transfer function

Characteristics :

- The output is measured and compared with the input.
- The difference (error) is used to adjust the input.
- A feedback path is present.

Advantages :

- More accurate
- Reduces effect of disturbances
- Less sensitive to parameter variations
- Self-correcting (error correction)

Disadvantages :

- More complex
- Costlier
- May become unstable if not properly designed

Note :

* If $H(s) = 1$ → Unity Feedback System
* If $H(s) \neq 1$ → Non-unity Feedback System

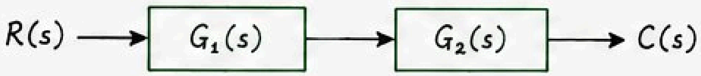
Key Point :

Closed loop systems are preferred in most practical applications due to higher accuracy and robustness.

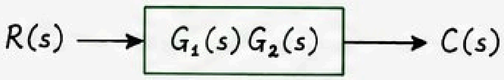
Block Diagram Algebra

Complex block diagrams can be reduced to simple equivalent block diagrams by using algebraic rules.

1. Series (Cascade) Connection



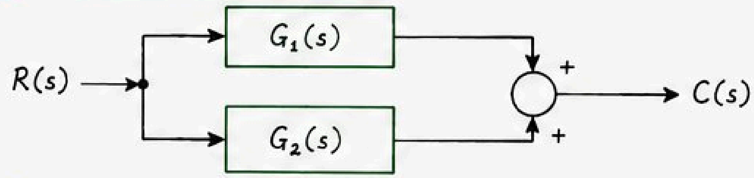
Equivalent block:



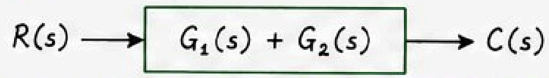
Overall transfer function:

$$G_{eq}(s) = G_1(s)\,G_2(s)$$

2. Parallel Connection



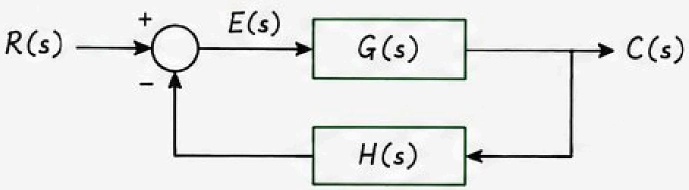
Equivalent block:



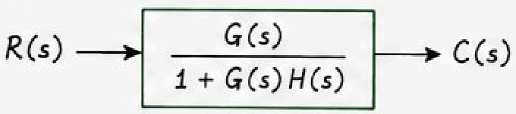
Overall transfer function:

$$G_{eq}(s) = G_1(s) + G_2(s)$$

3. Feedback Loop



Equivalent block:

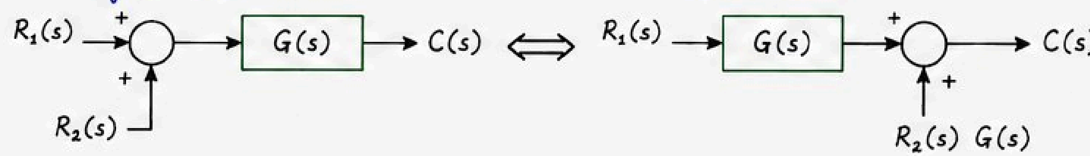


Overall transfer function:

$$G_{eq}(s) = \frac{G(s)}{1 + G(s)H(s)}$$

4. Moving Summing Point

(a) Summing point before $G(s)$ (b) Summing point after $G(s)$

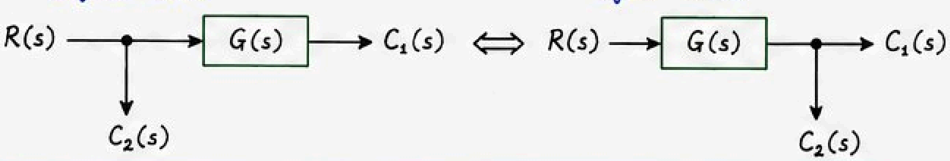


When moving summing point across a block $G(s)$:

- From left to right : Multiply the other input by $G(s)$
- From right to left : Divide the other input by $G(s)$

5. Moving Take-off Point

(a) Take-off point before $G(s)$ (b) Take-off point after $G(s)$

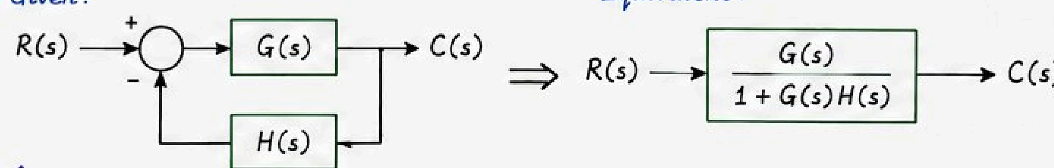


When moving take-off point across a block $G(s)$:

- From left to right : Multiply the taken-off signal by $G(s)$
- From right to left : Divide the taken-off signal by $G(s)$

6. Elimination of Feedback Loop

Given: Equivalent:



Notes:

1. All blocks are assumed to be linear time-invariant (LTI).
2. These rules are fundamental for simplifying block diagrams and finding overall transfer functions easily.